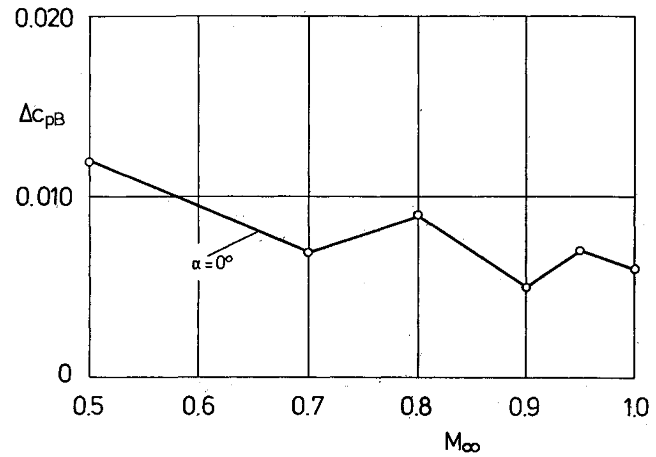

Fig. 1 The quantity Δc_{pB} as function of Mach number at $\alpha = 0$ deg.

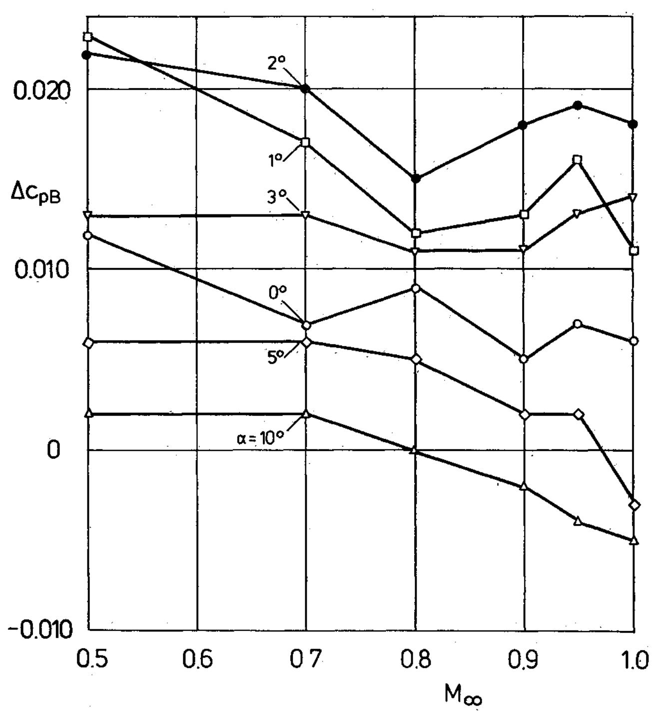

Fig. 2 The quantity Δc_{pB} as function of Mach number at several angles of incidence.

the cylinder diameter was $Re_D = 94{,}000$. The results show that Δc_{pB} depends on the depth of the cavity attaining a maximum value at $T/D \simeq 0.35$ with $\Delta c_{pB} \simeq 0.028$. The increase of the base pressure due to the cavity then decreases when T/D becomes larger. There was an approximately linear dependence between Δc_{pB} and T/D for $T/D > 0.35$. When one extrapolates the results of Morel up to $T/D = 1.20$, one gets $\Delta c_{pB} \simeq 0.014$, which agrees well with the value $\Delta c_{pB} = 0.012$, as measured by the present author for $M_\infty = 0.50$.

In Fig. 2, results are plotted for the angles of incidence $\alpha = 0$, 1, 2, 3, 5, and 10 deg. One can see that the base cavity at $\alpha = 1$ deg has a greater influence on the base pressure than at $\alpha = 0$ deg. The effect is again greatest for the Mach number $M_\infty = 0.50$ ($\Delta c_{pB} = 0.023$) and decreases with increasing Mach number. For $M_\infty = 1.00$, $\Delta c_{pB} = 0.011$ or approximately half the value at $M_\infty = 0.50$.

At the angle of incidence $\alpha = 2$ deg, the base cavity has the largest influence on the base pressure. The results in Fig. 2 show this clearly. The smallest value of Δc_{pB} ($= 0.015$) is attained at $M_\infty = 0.80$. All the other values are higher.

The results for $\alpha = 3$ deg show that the base cavity now has a smaller influence on the base pressure than for $\alpha = 2$ deg. At this angle of incidence, Δc_{pB} is practically independent of the Mach number.

At $\alpha = 5$ deg, the base cavity has only a small influence on the base pressure, which for $M_\infty = 0.50$ amounts to 3.5%. At the Mach number $M_\infty = 1.00$, the base pressure of the cavity base is smaller (and the base drag greater) than that of the normal base. The difference is very small, however, only about −1%.

For $\alpha = 10$ deg, one recognizes that at this angle of incidence, the base cavity has practically no more influence on the base pressure. This is also true for the results, which were obtained at the greater angles of incidence $\alpha = 15$, 20, and 25 deg. At $\alpha = 10$ deg and $M_\infty = 1.00$, $\Delta c_{pB} = -0.005$, which means that c_{pB} is 1.7% smaller for the cavity base than for the base without a cavity.

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New Approach to the Analysis and Control of Large Space Structures

G. Adomian*
General Analytics Corporation, Roswell, Georgia

Introduction

THE development of orbiting space stations will present very difficult problems of thermal and structural analysis because of the severe requirements posed by large sizes, low weight, high stiffness, and minimum mechanical and thermal distortion. Such distortions must be kept within very close tolerances of the order of millimeters.

The equations involved depend, of course, upon the particular designs, orbital positioning thrusts, atmospheric drag, gravity force, vibration, and heating and cooling from the sun, Earth, and electronic equipment. They will be nonlinear and, generally, stochastic as well because of uncertainties and thermally and mechanically induced vibrations. Usual solution methods leave much to be desired, as is well known. Recent work,[1–4] however, provides methods that can show analytic dependences, minimize computation, and provide physically realistic solutions not obtainable by usual methods, and are applicable to algebraic ordinary or partial differential and integrodifferential equation systems with composite nonlinearities, stochastic parameters, retarded effects, and complex initial-boundary conditions that can be nonlinear, stochastic, or even coupled. Applications to multidimensional nonlinear stochastic control problems relevant to large structures in orbit are being considered.

Our objective is realistic solution of the nonlinear systems of equations that arise in the modeling of such problems. Realistic solution means solution of the problem *as it is*, rather than changing the problem to make it easily solvable. Thus, perturbation, linearization, assumptions of weak nonlinearity, small

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*Senior Research Scientist.

fluctuations, and convenient stochastic processes must be justified, and used only when no other approach is possible. In systems involving stochastic parameters, e.g., differential equations with stochastic process coefficients—the stochastic operator case, usual analyses employ perturbation or hierarchy methods, which require that fluctuations be small, or assume a special nature or behavior for the processes—for mathematical not physical reasons. The literature abounds with unphysical assumptions and approximations, such as white noise, monochromatic approximation, local independence, etc. These limitations and assumptions are made for mathematical tractability and use of well-known theory. Yet, our final objective must not be simply the satisfaction of stating theorems and abstruse conditions, but to find solutions in close correspondence with actual physical behavior. Numerical results on supercomputers may lead to massive printouts that make dependences and relationships difficult to see. The solution sought should be that of the problem at hand, not one tailored to machine computation or the use of existing theorems. Thus, we propose to solve systems of multidimensional nonlinear stochastic partial differential equations in space and time—or ordinary differential equations or integrodifferential or delay differential equations (and special cases in which equations become linear or deterministic or one-dimensional)—without linearization, discretization, or perturbation. The method used herein is the decomposition method.[1,2]

Methodology

This method is an approximation method. It yields a series solution. It is not, however, to be viewed as less desirable than a closed-form analytical solution that has been arrived at by forcing the problem into a linear deterministic mold. Actually, it provides a better solution more in conformity with the physical situation. All modeling is approximate and a solution that provides, as this does, a rapidly converging continuous analytic approximation to the nonlinear problem (rather than a so-called exact solution to a linearized problem) may very well be more "exact."

We begin with the (deterministic) form $Fu = g(t)$, where F is a nonlinear ordinary differential operator with linear and nonlinear terms. The linear term is written as $Lu + Ru$, where L is invertible. To avoid difficult integrations, we choose L as the highest ordered derivative. R is the remainder of the linear operator. The nonlinear term is represented by Nu. Thus, $Lu + Ru + Nu = g$, and we write

$$Lu = g - Ru - Nu$$

$$L^{-1}Lu = L^{-1}g - L^{-1}Ru - L^{-1}Nu$$

For initial-value problems, we conveniently define L^{-1} for $L = \mathrm{d}^n/\mathrm{d}t^n$ as the n-fold definite integration operator from 0 to t. For the operator $L = \mathrm{d}^2/\mathrm{d}t^2$, for example, we have $L^{-1}Lu = u - u(0) - tu'(0)$ and, therefore,

$$u = u(0) + tu'(0) + L^{-1}g - L^{-1}Ru - L^{-1}Nu \qquad (1)$$

For the same operator but a boundary-value problem, we let L^{-1} be an indefinite integral and write $u = A + Bt$ for the first two terms and evaluate A and B from the given conditions. The first three terms of Eq. (1) are identified as u_0 in the assumed decomposition

$$u = \sum_{n=0}^{\infty} u_n$$

Finally, we write

$$Nu = \sum_{n=0}^{\infty} A_n(u_0, u_1, \ldots, u_n)$$

where the A_n are specially generated polynomials for the particular nonlinearity, which depend only on the u_0 to u_n components. We now have

$$u = \sum_{n=0}^{\infty} u_n = u_0 - L^{-1}R \sum_{n=0}^{\infty} u_n - L^{-1} \sum_{n=0}^{\infty} A_n$$

so that

$$u_1 = -L^{-1}Ru_0 - L^{-1}A_0$$

$$u_2 = -L^{-1}Ru_1 - L^{-1}A_1$$

etc., all components are determinable since A_0 depends only on A_0 depends only on u_0, A_1 depends only on u_0, u_1, etc. The practical solution will be the n-term approximation

$$\phi_n = \sum_{i=0}^{n-1} u_i$$

and

$$\lim_{n\to\infty} \phi_n = \sum_{i=0}^{\infty} u_i = u$$

by definition.

In the linear case where Nu vanishes, we have

$$u = u_0 - L^{-1}Ru_0 - L^{-1}Ru_1 - \cdots = u_0 - L^{-1}Ru_0 + (L^{-1}R)(L^{-1}R)u_0 - \cdots$$

or

$$u = \sum_{n=0}^{\infty} (-1)^n (L^{-1}R)^n u_0$$

If the given conditions are zero,

$$u_0 = L^{-1}g$$

and

$$u = \sum_{n=0}^{\infty} (-1)^n (L^{-1}R)^n L^{-1}g$$

Thus, $Fu = g$ becomes $u = F^{-1}g$, where the inverse is

$$F^{-1} = \sum_{n=0}^{\infty} (-1)^n (L^{-1}R)^n L^{-1}$$

Suppose now that $Fu = g$ is a partial differential equation, such as $\nabla^2 u + u_t = g$ with $g = g(x,y,z,t)$. Write

$$[L_x + L_y + L_z + L_t]u = g$$

where L_x, L_y, and L_z are $\partial^2/\partial x^2$, $\partial^2/\partial y^2$, and $\partial^2/\partial z^2$, respectively, and $L_t = \partial/\partial t$. We solve this exactly as before, with one change. We must solve for *each* linear operator term, invert, add, and divide by the number of equations to get a single equation. Thus, we have $[L_t + L_x + L_y + L_z]u = g$, from which we get four equations:

$$L_t u = g - L_x u - L_y u - L_z u$$

$$L_x u = g - L_y u - L_z u - L_t u$$

$$L_y u = g - L_z u - L_t u - L_x u$$

$$L_z u = g - L_t u - L_x u - L_y u$$

Applying L_t^{-1} to the first, L_x^{-1} to the second, etc., and supposing that the homogeneous solutions vanish, we get

$$u = L_t^{-1}g - L_t^{-1}(L_x + L_y + L_z)u$$
$$= L_x^{-1}g - L_x^{-1}(L_y + L_z + L_t)u$$
$$= L_y^{-1}g - L_y^{-1}(L_z + L_t + L_x)u$$
$$= L_z^{-1}g - L_z^{-1}(L_t + L_x + L_y)u$$

Adding and dividing by four,[1,2]

$$u = (1/4)\{L_t^{-1} + L_x^{-1} + L_y^{-1} + L_z^{-1}\}g$$
$$-(1/4)\{L_t^{-1}(L_x + L_y + L_z) + L_x^{-1}(L_y + L_z + L_t)$$
$$+L_y^{-1}(L_z + L_t + L_x) + L_z^{-1}(L_t + L_x + L_y)\}u$$

Define

$$u_0 = (1/4)\{L_t^{-1} + L_x^{-1} + L_y^{-1} + L_z^{-1}\}g$$

Replacing u by

$$\sum_{n=0}^{\infty} u_n,$$

identify

$$u_{n+1} = -(1/4)\{L_t^{-1}(L_x + L_y + L_z) + L_x^{-1}(L_y + L_z + L_t)$$
$$+L_y^{-1}(L_z + L_t + L_x) + L_z^{-1}(L_t + L_x + L_y)\}u_n$$

for $n \geq 0$. Since each component depends on the preceding components, all of them can be written in terms of u_0. Abbreviating the preceding bracketed quantity by $\{\cdot\}$, we have

$$u = \sum_{n=0}^{\infty} (-1)^n(1/4)^n\{\cdot\}^n(1/4)[L_t^{-1} + L_x^{-1} + L_y^{-1} + L_z^{-1}]\,g$$

Consequently, we can write $u = \mathscr{L}^{-1}g$ with $\mathscr{L}^{-1}$ defined by

$$\mathscr{L}^{-1} = \sum_{n=0}^{\infty} (-1)^n(1/4)^n\{\cdot\}^n(1/4)[L_t^{-1} + L_x^{-1} + L_y^{-1} + L_z^{-1}]$$

It has now been proved that we can include a nonlinear term $f(u)$ in the partial differential equation and find the solution in the same manner if, for $f(u)$, we write

$$\sum_{n=0}^{\infty} A_n.$$

As an example of partial differential equations, consider

$$\frac{\partial u}{\partial t} = x^2 - \frac{1}{4}\left(\frac{\partial u}{\partial x}\right)^2, \qquad u(x,0) = 0$$

(Note that, since u_x is in the nonlinear term, we have only one linear operator, so we do not get several equations and the procedure is the same as for an ordinary differential equation.) Writing $L_t = \partial/\partial t$, we have $L_t u = x^2 - (1/4)(u_x)^2$. The inverse

$$L_t^{-1} = \int_0^t [\cdot]\, dt$$

hence $L_t^{-1}L_t u = L_t^{-1}x^2 - (1/4)L_t^{-1}(u_x)^2$. Since the left-hand side is $u - u(0) = u$, we have

$$\sum_{n=0}^{\infty} u_n = u_0 - (1/4)L_t^{-1} \sum_{n=0}^{\infty} A_n$$

where we let

$$u = \sum_{n=0}^{\infty} u_n$$

identify $u_0 = L_t^{-1}x^2 = x^2t$, and replace the nonlinearity $(u_x)^2$ by the A_n polynomials. The A_n polynomials for u^2 are $A_0 = u_0^2$, $A_1 = 2u_0u_1$, $A_2 = u_1^2 + 2u_0u_2, \ldots$. (See, for example, Ref. 1.) Consequently,

$$u_1 = -(1/4)L_t^{-1}(u_{0_x})^2 = -(1/4)L_t^{-1}(4x^2t^2) = -x^2t^3/3$$

$$u_2 = -(1/4)L_t^{-1}(2u_{0_x}u_{1_x}) = (2/15)x^2t^5$$

$$u_3 = -(1/4)L_t^{-1}[u_{1_x}^2 + 2u_{0_x}u_{2_x}],$$

etc., so that

$$u = x^2\left(t - \frac{t^3}{3} + \frac{2}{15}t^5 - \cdots\right)$$

or

$$u = x^2 \tanh t$$

for $|t| < \pi/2$, which is easily verified not only for this solution but for the approximation

$$\sum_{i=0}^{n-1} u_i$$

for any n (as discussed in Ref. 2).

Now suppose our equations are coupled differential or partial differential equations. Suppose we have, for example, a system of equations in u and v. It is necessary only to define the pair u_0, v_0 and then find u_1, v_1 in terms of u_0, v_0, etc. For n equations, we have an n vector of terms for the first component. Then an n vector of second components is found in terms of the first. The procedure is discussed in detail in Ref. 2; we will only illustrate the procedure here.

Systems of Nonlinear Partial Differential Equations

Consider now a system of nonlinear partial differential equations given by $u_t = uu_x + vu_y$ and $v_t = uv_x + vv_y$ given $u(x,y,0) = f(x,y) = x + y$ and $v(x,y,0) = g(x,y) = x + y$. Let $L_t = \partial/\partial t$, $L_x = \partial/\partial x$, and $L_y = \partial/\partial y$. Then,

$$L_t u = uL_x u + vL_y u$$

$$L_t v = uL_x v + vL_y v$$

$$L_t^{-1} = \int_0^t [\cdot]\, dt$$

hence,

$$u = u(x,y,0) + L_t^{-1}uL_x u + L_t^{-1}vL_y u$$

$$v = v(x,y,0) + L_t^{-1}uL_x v + L_t^{-1}vL_y v$$

Let

$$u = \sum_{n=0}^{\infty} u_n \quad \text{and} \quad v = \sum_{n=0}^{\infty} v_n$$

and let $u_0 = u(x,y,0) = x + y$ and $v_0 = v(x,y,0) = x + y$ so that the first terms of u and v are known. We now have

$$u = u_0 + L_t^{-1}uL_x u + L_t^{-1}vL_y u$$

$$v = v_0 + L_t^{-1}uL_x v + L_t^{-1}vL_y v$$

We can use the A_n polynomials[1,2,4] for the nonlinear terms, thus†

$$u = u_0 + L_t^{-1} \sum_{n=0}^{\infty} A_n(uL_x u) + L_t^{-1} \sum_{n=0}^{\infty} A_n(vL_y u)$$

$$v = v_0 + L_t^{-1} \sum_{n=0}^{\infty} A_n(uL_x v) + L_t^{-1} \sum_{n=0}^{\infty} A_n(vL_y v)$$

$$A_0(uL_x u) = u_0 L_x u_0$$

$$A_1(uL_x u) = u_0 L_x u_1 + u_1 L_x u_0$$

$$A_2(uL_x u) = u_0 L_x u_2 + u_1 L_x u_1 + u_2 L_x u_0,$$

etc., for the other A_n. A simple rule here is the sum of the subscripts of each term is the same as the subscript of A. Consequently,

$$u_1 = L_t^{-1} u_0 L_x u_0 + L_t^{-1} v_0 L_y u_0$$

$$v_1 = L_t^{-1} u_0 L_x v_0 + L_t^{-1} v_0 L_y v_0$$

which yields the next component of u and v. Then,

$$u_2 = L_t^{-1}[u_0 L_x u_1 + u_1 L_x u_0] + L_t^{-1}[v_0 L_y u_1 + v_1 L_y u_0]$$

$$v_2 = L_t^{-1}[u_0 L_x v_1 + u_1 L_x v_0] + L_t^{-1}[v_0 L_y v_1 + v_1 L_y v_0],$$

etc., up to the following n-term approximations as our approximate solutions:

$$\sum_{i=0}^{n-1} u_i, \text{ for } u \qquad \sum_{i=0}^{n-1} v_i, \text{ for } v$$

Therefore, $u_0 = v_0 = x + y$, then u_1 and v_1 can be calculated as

$$u_1 = L_t^{-1} u_0 L_x u_0 + L_t^{-1} v_0 L_y u_0 = xt + yt + xt + yt = 2xt + 2yt$$

$$v_1 = L_t^{-1} u_0 L_x v_0 + L_t^{-1} v_0 L_y v_0 = 2xt + 2yt$$

†The notation $A_n(uL_x u)$ means the A_n generated for uu_x.

Continuing, we calculate $u_2 = 4t^2(x + y)$ and $v_2 = 4t^2(x + y)$, etc. Thus,

$$u = (x + y) + 2t(x + y) + 4t^2(x + y) + \cdots = (x + y)/(1 - 2t)$$

$$v = (x + y) + 2t(x + y) + 4t^2(x + y) + \cdots = (x + y)/1 - 2t)$$

The inclusion of stochastic processes is dealt with in Refs. 1 and 2 and elsewhere. The approximation ϕ_n becomes a stochastic series, and no statistical independence problems[1] are encountered in obtaining first- and second-order statistics from ϕ_n.

Conclusions

We have summarized the use of the decomposition method in some simple examples which can also be done by other methods for verification. It has been applied to some typical general vibration and heat problems in Ref. 3, and is now being applied as well to frontier problems for differential and partial differential equations with nonlinear and/or stochastic parameters, inputs, or conditions, e.g., to the nonlinear stochastic general control problem without the customary limiting approximations and assumptions.

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Notice to Subscribers

We apologize that this issue was mailed to you late. As you may know, AIAA recently relocated its headquarters staff from New York, N.Y. to Washington, D.C., and this has caused some unavoidable disruption of staff operations. We will be able to make up some of the lost time each month and should be back to our normal schedule, with larger issues, in just a few months. In the meanwhile, we appreciate your patience.